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CONTROL IN TECHNICAL SYSTEMS

Approaches to Optimizing Guidance Methods to High-Speed Intensively Maneuvering Targets. Part II. Analyzing the Capabilities of Different Ways to Optimize Guidance Methods

V. S. Verba[*,a], **V. I. Merkulov**[*,b], **and V. P. Kharkov**[**,c]

[*] *JSC Concern Vega, Moscow, Russia*
[**] *LLC NPO NaukaSoft, Moscow, Russia*
e-mail: [a]*vvs.msk@gmail.com,* [b]*mvipost41@gmail.com,* [c]*charkovvp@rambler.ru*
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Abstract—Based on the requirements for optimization methods of high-speed aircraft interception systems (see part I of the study), different ways to optimize guidance methods are analyzed. The capabilities of the classical theory of optimal control, as well as its modifications with local optimization and local optimization by the minimum of quadratic-biquadratic performance functionals, are considered at the qualitative level within the concept of inverse dynamics problems. In addition, the ways to optimize the information support of all approaches are assessed.

Keywords: statistical theory of optimal control, local optimization, quadratic-biquadratic performance functional, inverse dynamics problem, adaptive analog-discrete filtering

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1. INTRODUCTION

The military and technical perfection of guidance systems is largely determined by the ways to optimize control laws and their information support, representing the basis for their development. Currently, many optimization methods known match the requirements of an accurate and cost-efficient operation to different degrees [1]. Note those of the statistical theory of optimal control (STOC), which are used to design the best guidance systems jointly in terms of accuracy and control costs. These methods are based on the principle of minimizing quadratic performance functionals, which contain both control errors and energy costs of control implementation.

In this class of optimization ways, it is necessary to distinguish rather complex classical STOC methods, ensuring the optimality of guidance systems for the entire interception time [2–9], and simpler ones, ensuring their local optimality for each current time instant [2–4, 10–12].

In the practical development of complex technical systems for various purposes, design methods based on the concept of inverse dynamics problems (IDP) [13–18] are also used. Their peculiarity lies in the sufficiently simple consideration of different nonlinearities during the control design procedure.

Gradient methods are most widespread among the design ways neglecting the energy consumption of control signals [1, 19]. These methods optimize control laws in terms of different performance functionals having an extremum on the system operation interval.

Various modifications of Kalman filters are often used to optimize information systems [3, 4, 20–23].

In recent time, the so-called intelligent optimization methods based on neural network approaches [24, 25] have been gaining popularity in the design of systems operating under a priori uncertainty.

The analysis of the requirements for the optimization methods carried out in [26] demonstrated, first of all, the possibility of forming nonstationary guidance methods, the possibility of operation in a given domain of application conditions and constraints, and the possibility of implementation by the dynamic properties of the carrier and the capability of estimating the coordinates used in the guidance method. Note that the latter possibility can be assessed only based on the design results of particular guidance methods.

Practical ways to implement nonstationary guidance methods with the possibility of changing control and information priorities during flight, based on time-varying state models and coefficients of penalty matrices for accuracy and cost-efficient operation as functions of range and speed, were discussed in detail in [4].

The methods for designing guidance laws for high-speed aircraft (HSA) based on the classical STOC approach in the Letov–Kalman formulation and its local modification (with the mismatch between the dynamic properties of the target and the interceptor treated as disturbances) were also considered in [4].

Below, we qualitatively assess different ways to optimize guidance methods to HSA that match, to a greater or lesser extent, the requirements discussed in [26].

Note that the sections and formulas mentioned below have a double numbering system: the first digit corresponds to the part of the study whereas the second to the section or formula.

2. THE CLASSICAL OPTIMAL CONTROL THEORY IN THE LETOV–KALMAN FORMULATION: ANALYSIS OF CAPABILITIES

Consider the mathematical apparatus of the conventional statistical theory of optimal control [3–9] in its simplest form when applied to the problem under study. For an interceptor described by

$$\dot{\mathbf{x}}_{\text{int}}(t) = \mathbf{F}_{\text{int}}\mathbf{x}_{\text{int}}(t) + \mathbf{B}_{\text{int}}\mathbf{u}(t) + \boldsymbol{\xi}_{\text{int}}(t), \quad \mathbf{x}_{\text{int}}(0) = \mathbf{x}_{\text{int}0}, \tag{2.1}$$

intended for a target with a trajectory

$$\dot{\mathbf{x}}_{\text{tar}}(t) = \mathbf{F}_{\text{tar}}\mathbf{x}_{\text{tar}}(t) + \boldsymbol{\xi}_{\text{tar}}(t), \quad \mathbf{x}_{\text{tar}}(0) = \mathbf{x}_{\text{tar}0}, \tag{2.2}$$

under available measurements

$$\mathbf{z}(t) = \mathbf{H}\mathbf{x}(t) + \boldsymbol{\xi}_{\text{z}}(t), \quad \mathbf{x}(t) = \left[\mathbf{x}_{\text{tar}}^{\text{T}}(t) \quad \mathbf{x}_{\text{int}}^{\text{T}}(t)\right]^{\text{T}}, \tag{2.3}$$

this apparatus yields the optimal control law

$$\mathbf{u}(t) = \mathbf{K}^{-1}\mathbf{B}_{\text{int}}^{\text{T}}\mathbf{P}(t)[\hat{\mathbf{x}}_{\text{tar}}(t) - \hat{\mathbf{x}}_{\text{int}}(t)], \tag{2.4}$$

$$\dot{\mathbf{P}}(t) = -\mathbf{L} - \mathbf{F}_{\text{int}}^{\text{T}}\mathbf{P}(t) - \mathbf{P}(t)\mathbf{F}_{\text{int}} + \mathbf{P}(t)^{\text{T}}\mathbf{B}_{\text{int}}\mathbf{K}^{-1}\mathbf{B}_{\text{int}}^{\text{T}}\mathbf{P}(t), \quad \mathbf{P}(t_{\text{fin}}) = \mathbf{Q}, \tag{2.5}$$

in terms of the minimum value of the quadratic Letov–Kalman performance functional

$$I = M\left\{[\mathbf{x}_{\text{tar}}(t_{\text{fin}}) - \mathbf{x}_{\text{int}}(t_{\text{fin}})]^{\text{T}}\mathbf{Q}[\mathbf{x}_{\text{tar}}(t_{\text{fin}}) - \mathbf{x}_{\text{int}}(t_{\text{fin}})] + \int_0^{t_{\text{fin}}}[\mathbf{x}_{\text{tar}}(t) - \mathbf{x}_{\text{int}}(t)]^{\text{T}}\mathbf{L}[\mathbf{x}_{\text{tar}}(t) - \mathbf{x}_{\text{int}}(t)]dt + \int_0^{t_{\text{fin}}}\mathbf{u}^{\text{T}}(t)\mathbf{K}\mathbf{u}(t)dt\right\}. \tag{2.6}$$

Here, $\mathbf{x}_{\text{int}}$ and $\mathbf{x}_{\text{tar}}$ are the n-dimensional vectors of interceptor and target states, respectively; $\mathbf{F}_{\text{int}}$ and $\mathbf{F}_{\text{tar}}$ are the internal connection matrices of the processes (2.1) and (2.2); $\mathbf{u}$ is the r-dimensional control vector ($r \leqslant n$); $\mathbf{B}_{\text{int}}$ is the control efficiency matrix; $\mathbf{z}$ is the m-dimensional measurement vector ($m \leqslant 2n$); $\mathbf{H}$ is the connection matrix of (2.1) and (2.2) with (2.3); $\mathbf{P}$ is a symmetric matrix defining the current weight of the control errors; t and t_{fin} are the current and final control time, respectively; $\mathbf{Q}$ and $\mathbf{L}$ are nonnegative definite penalty matrices for the final and current control accuracy, respectively; $\mathbf{K}$ is a positive definite penalty matrix for control signals; $\boldsymbol{\xi}_{\text{int}}$, $\boldsymbol{\xi}_{\text{tar}}$, and $\boldsymbol{\xi}_{\text{z}}$ are the vectors of centered Gaussian noises of the state and measurement vectors, respectively; finally, $\hat{\mathbf{x}}_{\text{int}}$ and $\hat{\mathbf{x}}_{\text{tar}}$ are the vectors of the optimal estimates of the processes (2.1) and (2.2), respectively.

Note that the minimized functional (2.6) includes three terms. The first, terminal, defines the system accuracy at the final time instant of control; the second, the integral accuracy over the entire control time; and the third, the energy consumption of control signals. Minimizing the performance functional (2.6), the control law (2.4), (2.5) is jointly the best in terms of accuracy and cost efficiency, which is an undoubted advantage.

Direct analysis of (2.1)–(2.5) leads to several general conclusions.

1. The control law (2.4), (2.5) depends on the system state $\hat{\mathbf{x}}_{\text{int}}$ and $\hat{\mathbf{x}}_{\text{tar}}$, its capability to perceive control signals (determined by the matrix $\mathbf{B}_{\text{int}}$), the penalties $\mathbf{K}$ for control signals, and the weight matrix $\mathbf{P}$. The larger the control penalty is, the smaller the signals $\mathbf{u}$ will be (accordingly, the more cost-efficient the system will be but the less accurate). The latter is due to that small values of $\mathbf{u}$ cause small values of $\dot{\mathbf{x}}_{\text{int}}$ in (2.1), and consequently small targeted changes of $\mathbf{x}_{\text{int}}$. If system (2.1) perceives the control signals $\mathbf{u}$ well (the matrix $\mathbf{B}_{\text{int}}$ has large coefficients), then it is reasonable to make them large: in this case, there will be large values of $\dot{\mathbf{x}}_{\text{int}}$ and the system will quickly change its state $\mathbf{x}_{\text{int}}$. If the coefficients of the matrix $\mathbf{B}_{\text{int}}$ are small, then large control signals should not be used, as it will result in unreasonably high energy consumption with very little gain in accuracy.

2. In (2.4), the coefficients of the matrix $\mathbf{P}$ aggregately describe the current accuracy and cost efficiency penalties determined by the matrices $\mathbf{L}$ and $\mathbf{K}$, the deterministic connections, and the efficiency of the control signals conditioned by the matrices $\mathbf{F}_{\text{int}}$ and $\mathbf{B}_{\text{int}}$. The impact of deterministic connections is manifested in that a change in the penalty l_{ii} for the operation accuracy on some coordinate x_i causes a change in the accuracy on other coordinates functionally connected with x_i. The corresponding changes in the matrix $\mathbf{P}$ lead to changes in control signals and, consequently, in the system's efficiency.

3. The peculiarity of (2.4), (2.5) is that the coefficients of the matrix (2.5) are calculated in inverse time from t_{fin} to t when solving the Riccati equation, while in (2.4) they are used in direct time. Note that, due to the number of equations in (2.5) to be solved for determining the matrix $\mathbf{P}$, the control formation complexity significantly exceeds that of the optimized system (2.1). Moreover, even a slight increase in the dimension of (2.1) leads to excessively many equations to be solved when calculating the matrix $\mathbf{P}$. (The number of the equations is n^2.) This phenomenon, called the curse of dimensionality and characteristic of many optimal systems, restrains the application of optimal control algorithms for high-dimensional complex systems. However, for time-invariant systems, the matrix $\mathbf{P}$, defined only by a priori information, can be obtained in advance. Accordingly, the coefficients $\mathbf{K}^{-1}\mathbf{B}_{\text{int}}^{\text{T}}\mathbf{P}(t)$ for (2.4), whose number is defined by the dimension $r \times n$, can also be calculated in advance. Hence, the procedure of using (2.4), (2.5) becomes somewhat simpler in practice.

4. Assigning different penalties $\mathbf{L}$ and $\mathbf{Q}$ to the current and final accuracy allows implementing different errors at different operation stages of the interception system, thereby ensuring the desired accuracy at the end of control under very low current energy costs.

For the problem of designing a guidance method to HSA with the apparatus (2.1)–(2.6), we draw the following conclusions:

- By adjusting the form the matrices $\mathbf{Q}$, $\mathbf{L}$, and $\mathbf{K}$, using the representation of their elements as a function of the state coordinates, it is possible to form a time-varying guidance law [4] with the redistribution of control functions depending on the value of the state coordinates with a significant complication of the calculation procedure of (2.5).
- The linear dependence of (2.4) on the control errors ($\Delta\mathbf{x} = \mathbf{x}_{\text{tar}} - \mathbf{x}_{\text{int}}$) does not provide an enhanced role for control to withdraw from the stability loss bounds.
- To implement (2.4), (2.5), the guidance time should be known, which is practically impossible.
- When linear (linearized) state models are used, the control law (2.4), (2.5) ensures acceptable universality of the guidance method, implementing stable operation in a wide range of application conditions [3].
- The need to solve the high-dimensional two-point boundary value problem due to resolving equation (2.5) in reverse time from t_{fin} to t, while the control law (2.4) is formed in direct time from t to t_{fin}, significantly complicates the control design procedure.
- In the law (2.4), the dependence of the control signal on the dynamic properties of the interceptor ($\mathbf{F}_{\text{int}}$) is considered in a complex way when solving (2.5), which makes it difficult to predict its significance in interception problems.
- The capability to form all the optimal estimates $\hat{\mathbf{x}}_{\text{int}}$ and $\hat{\mathbf{x}}_{\text{tar}}$, required to implement (2.4), is determined by the observability condition [3], see Section 6. The fulfillment of this condition depends on the internal connections of (2.1), (2.2) and the set of measurement sensors in (2.3).

In conclusion, we underline that the classical optimal control theory in the Letov–Kalman formulation does not satisfy all the requirements for implementing the guidance method to HSA.

The most challenging obstacles to optimizing interception methods using this method are the need to know the guidance time and the complexity of solving the two-point boundary value problem.

3. LOCAL OPTIMIZATION METHODS BY THE MINIMUM OF QUADRATIC PERFORMANCE FUNCTIONALS: ANALYSIS OF CAPABILITIES

It seems more promising to use local optimization methods that minimize performance functionals for each current time instant without requiring knowledge of the guidance time. Then it is possible to generate control without solving the complex two-point boundary value problem, which significantly simplifies the control design procedure. Moreover, within this approach, various disturbances affecting the interceptor can be quite simply considered directly in the control law without expanding the state vector. In this case, for an interceptor described by

$$\dot{\mathbf{x}}_{\text{int}}(t) = \mathbf{F}_{\text{int}}\mathbf{x}_{\text{int}}(t) + \mathbf{B}_{\text{int}}\mathbf{u}(t) + \mathbf{s}_{\text{int}}(t) + \boldsymbol{\xi}_{\text{int}}(t), \quad \mathbf{x}_{\text{int}}(0) = \mathbf{x}_{\text{int}0}, \tag{3.1}$$

intended for an HSA with the trajectory (2.2) under the available measurements (2.3), the local optimization method [3] yields the optimal control law

$$\mathbf{u}(t) = \mathbf{K}^{-1}\mathbf{B}_{\text{int}}^{\mathrm{T}}\left[\mathbf{Q}(\hat{\mathbf{x}}_{\text{tar}}(t) - \hat{\mathbf{x}}_{\text{int}}(t)) - \mathbf{G}\hat{\mathbf{s}}_{\text{int}}(t)\right], \tag{3.2}$$

in terms of the minimum value of the performance functional

$$I = M\left\{[\mathbf{x}_{\text{tar}}(t) - \mathbf{x}_{\text{int}}(t)]^{\mathrm{T}}\mathbf{Q}[\mathbf{x}_{\text{tar}}(t) - \mathbf{x}_{\text{int}}(t)] \right.$$
$$\left. + 2[\mathbf{x}_{\text{tar}}(t) - \mathbf{x}_{\text{int}}(t)]^{\mathrm{T}}\mathbf{G}\mathbf{s}_{\text{int}}(t) + \mathbf{s}_{\text{int}}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{s}_{\text{int}}(t) + \int_0^t \mathbf{u}^{\mathrm{T}}(t)\mathbf{K}\mathbf{u}(t)dt\right\}. \tag{3.3}$$

Here, $\mathbf{s}_{\rm int}$ and $\hat{\mathbf{s}}_{\rm int}$ are the n-dimensional vectors of the measured disturbances affecting the interceptor and their optimal estimates, respectively; $\mathbf{G}$ is a nonnegative matrix defining the weight of disturbances in the control law (3.2).

For the problem of designing a guidance method to HSA [26], the analysis of (3.1)–(3.3) leads to the following conclusions:

1. Using the coefficients of the matrices $\mathbf{Q}$ and $\mathbf{G}$ as functions of the state coordinates (the range and approach speed), it is possible to generate control laws [4] with rather easily assigned instants of, first, changing control priorities during guidance and surveillance trajectory control [4] and, second, adjusting the effects of disturbances on different interception trajectory sections.

2. The method has high implementability due to the following features:

- the capability to optimize the guidance system for a particular carrier by forming additional correction signals that compensate for its inertia;
- the capability to consider a wide range of natural and virtual disturbances [3] in the form of mismatches between the dynamic properties of the target and interceptor, the prediction of the target's spatial position compensating for the carrier's inertia, the approach of the state coordinates to acceptable stability loss bounds, etc.;
- the simplicity of control signals formation for each current time instant (there is no need to know the guidance time and solve the complex two-point boundary value problem).

The capability of estimating all state coordinates used in the guidance method can be determined only by the results of designing particular control laws.

3. The method possesses high universality, characterized by the capability to design stable guidance methods in a wide field of application conditions, including those not corresponding to the models underlying the design procedure [4].

4. The method does not guarantee withdrawal from the stability loss bounds due to the linear dependence of (3.2) on the control errors.

4. LOCAL OPTIMIZATION METHODS BY THE MINIMUM OF QUADRATIC-BIQUADRATIC PERFORMANCE FUNCTIONALS: ANALYSIS OF CAPABILITIES

According to the capabilities of guidance design schemes with local optimization methods by the minimum of conventional quadratic performance functionals [4] (see the above analysis), the problems of withdrawing the carrier from stability loss bounds and using the derivatives of the line-of-sight (LoS) angle rate in control laws are still challenging, which significantly complicates their information support procedure.

Both problems can be solved by applying guidance methods with a nonlinear (cubic) dependence on the control errors generated during the local minimization of quadratic-biquadratic performance functionals [3, 27].

In the elementary case, for the interceptor (2.1) intended for a target with the trajectory (2.2) under the available measurements (2.3), this approach yields the optimal control law

$$\mathbf{u}(t) = \mathbf{K}^{-1}\mathbf{B}_{\rm int}^{\rm T}\left\{\mathbf{Q} + 2\left[\Delta\hat{\mathbf{x}}(t)\Delta\hat{\mathbf{x}}(t)^{\rm T}\mathbf{R}\right]\right\}\Delta\hat{\mathbf{x}}(t), \quad \Delta\hat{\mathbf{x}}(t) = \hat{\mathbf{x}}_{\rm tar}(t) - \hat{\mathbf{x}}_{\rm int}(t), \tag{4.1}$$

in terms of the minimum value of the performance functional

$$I = M\left\{\Delta\mathbf{x}(t)^{\rm T}\mathbf{Q}\Delta\mathbf{x}(t) + \mathbf{x}(t)^{\rm T}\left[\Delta\mathbf{x}(t)\Delta\mathbf{x}(t)^{\rm T}\mathbf{R}\right]\Delta\mathbf{x}(t) + \int\limits_0^t \mathbf{u}^{\rm T}(t)\mathbf{K}\mathbf{u}(t)dt\right\}. \tag{4.2}$$

Direct analysis of (4.1) brings to the following conclusions.

1. The control signal consists of two parts: the first

$$\mathbf{K}^{-1}\mathbf{B}_{\text{int}}^{\text{T}}\mathbf{Q}\Delta\hat{\mathbf{x}}(t) \tag{4.3}$$

determines its linear component whereas the second

$$2\mathbf{K}^{-1}\mathbf{B}_{\text{int}}^{\text{T}}\left[\Delta\hat{\mathbf{x}}(t)\Delta\hat{\mathbf{x}}(t)^{\text{T}}\mathbf{R}\right]\Delta\hat{\mathbf{x}}(t) \tag{4.4}$$

the cubic component. In addition to the terms proportional to Δx_i^3 $(i = \overline{1,n})$, formula (4.4) contains the combined terms $\Delta x_i^2\Delta x_j$ and $\Delta x_i\Delta x_j^2$ $(i = \overline{1,n},\ j = \overline{1,n},\ i \neq j)$.

2. The relationships between (4.3) and (4.4) depend on the coefficients of the matrices $\mathbf{Q}$ and $\mathbf{R}$ and, moreover, on the relationship between the control errors Δx_i and Δx_j.

For small errors ($\Delta x_i \to 0$), the cubic component has almost no effect on the control signal and guidance accuracy, ensuring the high sensitivity of (4.1) to small errors.

Under large values of Δx_i, the cubic component becomes predominant, accelerating the handling of dangerous control errors.

The resulting control laws [4, 27] do not require estimating the derivatives of the LoS angle rate; the role of such estimates is played, to some extent, by the terms $\Delta x_i^2\Delta x_j$ and $\Delta x_i\Delta x_j^2$.

3. By manipulating the particular composition of the coefficients in the matrices $\left[\Delta\mathbf{x}(t)\Delta\mathbf{x}(t)^{\text{T}}\right]$ and $\mathbf{R}$, one can obtain different control laws with different sets of the combinational components.

4. Using the local optimization method by the performance functional (4.2) allows preserving all its advantages discussed in Section 3.

According to the analysis results [3–9], the mathematical apparatus of the local version of the statistical theory of optimal control with minimizing quadratic-biquadratic performance functionals is preferable to satisfy the set of requirements listed in [26]. This apparatus implements a wide range of control laws jointly the best in terms of accuracy and control costs.

5. INVERSE DYNAMICS PROBLEMS

There exists an entire class of control problems in which the control design procedure cannot be reduced to minimizing a certain rigorously defined performance functional. In particular, the matter concerns control problems with a natural global criterion reflecting, correctly and completely, the problem content. Here, the control objective is often to maintain definite relations between certain components of the state vector of plants. These relations usually describe the normal operation conditions of the plant or the nature of its transients.

Recently, control design methods based on the concept of inverse dynamics problems (IDP) have become widespread to solve such problems [13, 14]. The problem of implementing some model reference trajectory of a controlled system was among the first research works that underlined the development of the IDP method [15]. There are several known techniques and methods for solving control problems based on this method [16–18]. As was demonstrated in [18], the structural properties of control algorithms for linear systems are completely identical to those of the algorithms obtained by the classical analytical design theory with quadratic performance functionals.

Consider the IDP method to determine whether it meets the requirements for control design methods in guidance problems [26].

Let the mathematical model of a controlled observable dynamic system be described by a differential operator

$$\dot{\mathbf{x}}_{\text{int}}(t) = \mathbf{f}(\mathbf{x}_{\text{int}}(t), \mathbf{u}(t), \mathbf{a}(t), \mathbf{s}_{\text{int}}(t), t), \tag{5.1}$$

where $\mathbf{x}_{\text{int}}(t) = [x_{\text{int}1}(t), \ldots, x_{\text{int}n}(t)]^{\text{T}}$ denotes the n-dimensional state vector of the system;

$\mathbf{a}(t) = [a_1(t), \ldots, a_p(t)]^{\text{T}}$ is the p-dimensional vector of parameters;

$\mathbf{u}(t) = [u_1(t), \ldots, u_r(t)]^{\text{T}}$ stands for the r-dimensional vector of control functions;

$\mathbf{s}_{\text{int}}(t) = [s_{\text{int}1}(t), \ldots, s_{\text{int}n}(t)]^{\text{T}}$ is the n-dimensional vector of controlled exogenous disturbances representing a given time-varying function from the space L_2.

By assumption, the vector function $\mathbf{f}(\mathbf{x}_{\text{int}}(t), \mathbf{u}(t), \mathbf{a}(t), \mathbf{s}_{\text{int}}(t), t)$ is continuous and differentiable with respect to all the variables $\mathbf{x}_{\text{int}}, \mathbf{u}, \mathbf{a}, \mathbf{s}_{\text{int}}$.

It is required to find a control law $\mathbf{u}(t)$ optimizing the performance functional

$$I = \int_{t_0}^{t} L(\mathbf{x}_{\text{int}}(t), \mathbf{x}_{\text{tar}}(t), \mathbf{u}(t), \mathbf{s}_{\text{int}}(t), t)dt, \tag{5.2}$$

where $L(\mathbf{x}_{\text{int}}(t), \mathbf{x}_{\text{tar}}(t), \mathbf{u}(t), \mathbf{s}_{\text{int}}(t), t)$ is a scalar nonnegative function. The terminal time t (control termination) can be given or arbitrary. The coordinates $\mathbf{x}_{\text{int}}(t)$ and $\mathbf{x}_{\text{tar}}(t)$ must satisfy some hypersurface constraints [16]:

$$\mathbf{C}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}}) = 0. \tag{5.3}$$

If the relation (5.3) fails as the result of disturbances or nonzero initial conditions, the controlled object, due to its inertia, will tend to this hypersurface according to the expression

$$\lim_{t\to\infty} \mathbf{C}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}}) = 0. \tag{5.4}$$

In formulas (5.3) and (5.4), the function $\mathbf{C}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}})$ is an r-dimensional vector function with a continuous derivative with respect to its arguments and ∞ indicates the transient time of the plant.

In the general case, the law of vanishing of the function $\mathbf{C}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}})$ in (5.4) can be considered the solution of the equation

$$\boldsymbol{\psi}_1\left[\lambda_i, \dot{\mathbf{C}}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}}), \ddot{\mathbf{C}}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}}), \ldots, \mathbf{C}^{(k)}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}})\right] = \boldsymbol{\psi}_2\left[\beta, \dot{\mathbf{C}}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}})\right], \quad i = \overline{1, k}, \tag{5.5}$$

where λ_i and β are arbitrary constant numbers making the solution of (5.5) stable; i corresponds to the number of the constant coefficient specifying the weight of the kth derivative of the constraint (5.3) or (5.4); finally, $\boldsymbol{\psi}_1[\bullet]$ and $\boldsymbol{\psi}_2[\bullet]$ are r-dimensional (generally nonlinear) vector functions. However, in many engineering applications, the functions $\boldsymbol{\psi}_1[\bullet]$ and $\boldsymbol{\psi}_2[\bullet]$ in equation (5.5) can be described by the relations

$$\begin{aligned}\boldsymbol{\psi}_1[\bullet] &= \mathbf{C}^{(k)}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}}) + \lambda_{k-1}\mathbf{C}^{(k-1)}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}}) + \ldots + \lambda_1\dot{\mathbf{C}}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}}),\\ \boldsymbol{\psi}_2[\bullet] &= \beta_0\mathbf{C}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}}) \quad \text{or} \quad \boldsymbol{\psi}_2[\bullet] = \beta_0\mathbf{C}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}}) + \beta_2\mathbf{C}^3(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}}).\end{aligned} \tag{5.6}$$

The coordinates of the n-dimensional vector $\mathbf{x}_{\text{tar}}(t)$ by their physical nature either coincide with the vector $\mathbf{x}_{\text{int}}(t)$ or represent some combination of its components.

The control $\mathbf{u}(t)$ must be defined as a function of the state coordinates of system (5.1) and the coordinates of the desired trajectory. The interception problem can be represented as the collection of two motions in the horizontal and vertical planes. Therefore, we consider the application of the control design method in a single plane under a single (scalar) control signal.

Scalar control action

The exogenous disturbance $\mathbf{s}_{\text{int}}(t)$ in (5.1) is a given time-varying function, and all its components are controlled.

Consider the case when system (5.1) can be written as the system of linear equations

$$\dot{\mathbf{x}}(t) = \mathbf{F}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{s}_{\mathrm{int}}(t), \tag{5.7}$$

where $\mathbf{F} = \|f_{ij}\|$ is a square matrix of dimensions $n \times n$ with known elements; $\mathbf{B}$ is the column vector of control coefficients in each system equation; the exogenous disturbance $\mathbf{s}_{\mathrm{int}}(t)$ is a given time-varying function, and all its components are controlled; finally, $u(t)$ is the scalar control action.

Note that the number of controlled coordinates of the vector $\mathbf{x}(t)$ in the steady-state mode (hence, the dimension of the vector $\mathbf{C}(\mathbf{x}_{\mathrm{int}}, \mathbf{x}_{\mathrm{tar}})$) is determined by the dimension of the control vector. Then, without loss of generality, system (5.7) can be supposed equivalent to the scalar differential equation

$$\dot{x}^{(n)}_{\mathrm{int}1}(t) + \sum_{i=0}^{n-1} \alpha_i x^{(i)}_{\mathrm{int}1}(t) = \sum_{j=0}^{r} b_j u^{(j)}(t) + \sum_{l=0}^{n} k_l s_{\mathrm{int}l}(t), \tag{5.8}$$

where $x_1(t)$ is the output coordinate of system (5.7); k_l is the weight of the disturbances $s_{\mathrm{int}l}$.

For the sake of definiteness, let the function $\mathbf{C}(\mathbf{x}_{\mathrm{int}}, \mathbf{x}_{\mathrm{tar}})$ have the form

$$\mathbf{C}(\mathbf{x}_{\mathrm{int}}, \mathbf{x}_{\mathrm{tar}}) = x_{\mathrm{int}1}(t) - x_{\mathrm{tar}1}(t). \tag{5.9}$$

The solution of this problem will be found from the condition that $\mathbf{C}(\mathbf{x}_{\mathrm{int}}, \mathbf{x}_{\mathrm{tar}})$ vanishes according to some law:

$$\lim_{t\to\infty} \mathbf{C}(\mathbf{x}_{\mathrm{int}}, \mathbf{x}_{\mathrm{tar}}) = 0.$$

Moreover, the law $\mathbf{C}(\mathbf{x}_{\mathrm{int}}, \mathbf{x}_{\mathrm{tar}})$ can be defined by any differential operator, e.g., (5.6):

$$\mathbf{C}^{(n)}(\mathbf{x}_{\mathrm{int}}, \mathbf{x}_{\mathrm{tar}}) + \lambda_{n-1} \times \mathbf{C}^{(n-1)}(\mathbf{x}_{\mathrm{int}}, \mathbf{x}_{\mathrm{tar}}) + \ldots + \lambda_0 \times \mathbf{C}(\mathbf{x}_{\mathrm{int}}, \mathbf{x}_{\mathrm{tar}}) = 0, \tag{5.10}$$

where λ_j, $j = 0, 1, \ldots, n-1$, are any positive numbers making system (5.6) stable.

Substituting (5.8) into (5.10) yields the following rth-order differential equation for $u(t)$:

$$b_r u^{(r)}(t) + b_{r-1} u^{(r-1)}(t) + \ldots + b_0 u(t) = z(t), \tag{5.11}$$

where $z(t) = \sum_{i=0}^{n-1} \alpha_i x^{(i)}_{\mathrm{int}1}(t) - \sum_{j=0}^{n-1} \lambda_j (x_{\mathrm{int}1} - x_{\mathrm{tar}1}) + x^{(n)}_{\mathrm{tar}1}(t) - \sum_{l=0}^{n} k_l s_{\mathrm{int}l}(t)$.

The most interesting case is when the components of the vector $\mathbf{B}$ in equation (5.7) equal zero, except the last one. Then the control action satisfying (5.11) is given by

$$\begin{aligned} u(t) = b_0^{-1} &\left[-\sum_{j=0}^{n-1} \lambda_j x^{(j)}_{\mathrm{int}1}(t) + \sum_{i=0}^{n-1} \alpha_i x^{(i)}_{\mathrm{int}1}(t) + \sum_{j=0}^{n-1} \lambda_j x^{(j)}_{\mathrm{tar}1}(t) \right] \\ &- b_0^{-1} \left[\sum_{l=0}^{n} k_l s_{\mathrm{int}l}(t) - x^{(n)}_{\mathrm{tar}1}(t) \right]. \end{aligned} \tag{5.12}$$

In this expression, $\lambda_n = 1$.

Consider in detail the peculiarities of the controlled process under the control action given by equation (5.11) or (5.12). Let the parameters α_j, $j = 0, 1, \ldots, n-1$, and b_i, $i = 0, 1, \ldots, r$, of system (5.7) be known exactly; then the controlled process satisfies the equation

$$x^{(n)}_{\mathrm{int}1}(t) + \sum_{j=0}^{n-1} \lambda_j x^{(j)}_{\mathrm{int}1}(t) = \lambda_0 x_{\mathrm{int}1}(t) + \sum_{j=0}^{n-1} \lambda_j x^{(j)}_{\mathrm{tar}1}(t). \tag{5.13}$$

In matrix form, it can be written as

$$\dot{\mathbf{x}}_{\text{int}}(t) = \mathbf{A}_\lambda \mathbf{x}(t) + \mathbf{B}_\lambda \mathbf{x}_{\text{tar}}(t). \tag{5.14}$$

Due to (5.13) and (5.14), the properties of the controlled process are uniquely determined by the coefficients λ_j regardless of the properties of the original system. The reason is that $\mathbf{A}_\lambda$ represents a Frobenius matrix with the last row defined by the coefficients $\lambda_j,\ j = 0, 1, \ldots, n-1$.

The unknown coefficients $\lambda_j,\ j = 0, 1, \ldots, n-1$, are obtained from the necessary conditions for the optimality of the performance functional (5.2).

Note in conclusion that the control $u(t)$ designed by this method is a function of the state coordinates $\mathbf{x}_{\text{int}}$ and $\mathbf{x}_{\text{tar}}$ and the parameters $\lambda_j,\ j = 0, 1, \ldots, n-1$. In addition, for the nonlinear system, the mathematical model of the controlled process is also described by equation (5.14), i.e., the equation of the desired process.

The analysis of this control design method as applied to HSA interception shows the following:

— First, it allows assessing the possibility of forming control laws for both fixed and current guidance time.

— Second, the IDP method allows designing both linear and nonlinear laws by using different functions: $\boldsymbol{\psi}_2[\bullet] = \beta_0 \mathbf{C}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}})$ for linear control and $\boldsymbol{\psi}_2[\bullet] = \beta_0 \mathbf{C}(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}}) + \beta_2 \mathbf{C}^3(\mathbf{x}_{\text{int}}, \mathbf{x}_{\text{tar}})$ for linear cubic control. As a result, the carrier is withdrawn from stability loss bounds.

— Third, the control laws can be either time-invariant or time-varying, which is determined by the mathematical models (5.1)–(5.7) and the type of target maneuvering, stemming from the control laws (5.11) and (5.12).

— Fourth, in view of (5.14), the closed-loop control system has the required properties regardless of the control law: it is described by a linear differential equation and provides the achievable requirements for stability, overshoot, and robustness to a priori errors of no more than 30%.

The method implements target interception in a given fixed time if the interception trajectory is represented as a straight line and an arc with a known radius.

In addition, the control laws designed by the IDP method require reasonable computational resources for their real-time implementation.

6. ANALYSIS OF CAPABILITIES TO OPTIMIZE THE INFORMATION SUPPORT OF GUIDANCE METHODS

Information support, which reduces estimating the relative and absolute motion coordinates of the target and interceptor motion in guidance methods, is a prerequisite of their implementation [26]. According to the review of guidance methods [2, 4, 27], it is generally required to obtain the following estimates in each control plane: range, approach speed, target's relative bearing, target's LoS angle rate, and its derivatives.

The necessary conditions determining the capability of forming these estimates are given by the observability criterion [2, 3] based on the relationships between the original state models (2.1), (2.2), and (2.3). As applied to linear systems, this criterion has the form

$$\operatorname{rank}\left[\mathbf{H}^{\mathrm{T}} \middle| \mathbf{F}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}} \middle| \left(\mathbf{F}^{\mathrm{T}}\right)^2 \mathbf{H}^{\mathrm{T}} \middle| \ldots \middle| \left(\mathbf{F}^{\mathrm{T}}\right)^{N-1} \mathbf{H}^{\mathrm{T}} \right] = N = 2n, \tag{6.1}$$

where $\mathbf{F}$ is the dynamic matrix of the generalized state vector used in (2.3).

The physical meaning of (6.1) is that N independent equations with N unknowns can be obtained based on (2.1), (2.2), and (2.3) to relate the measurements to the estimates uniquely.

In the applied sense, along with establishing the very capability of designing filtering algorithms, the criterion (6.1) allows determining the set of measurement sensors to estimate the state vector. In addition, due to (6.1), the zero derivatives of the state vector should at least be measured to solve this problem [4]. As applied to the guidance problem, the range and relative bearing of the target should at least be measured.

Note that different state coordinates contribute differently to the guidance errors. As demonstrated by the studies [2, 28], the estimation errors of angular coordinates affect the homing accuracy by an order of magnitude or more than the estimation errors of the range and its derivatives.

Various approaches to selecting the information support optimization procedure are possible, depending on the antenna system.

The first approach is based on adaptive analog-discrete filtering algorithms.

The second involves multistage filtering.

The third one is to transform input signals in order to optimize the use of a given mechanically controlled antenna actuator.

The fourth approach is based on tracking systems with a nonlinear dependence on tracking errors that optimizes them by the minimum of a local quadratic-biquadratic performance functional. For details, see Section 4.

The theoretical foundations of these approaches, as well as particular estimation algorithms and research results concerning their effectiveness, were discussed in detail in [3, 4, 28].

There exist three reasons for using analog-discrete filtering:

— the necessity to generate a trajectory control signal for the interceptor continuously;

— the discrete and non-simultaneous arrival of measurements generated by sensors of various physical nature (e.g., radars and airborne signal systems) at the estimation algorithm;

— the unavailability of the state models reflecting adequately the complex spatial maneuvers of HSA, which predetermines the divergence of traditional Kalman filters and the necessity to apply different adaptation schemes.

Analog-discrete filtering includes extrapolation carried out with a small step τ, approaching the analog prediction by accuracy and providing a continuous control signal formation mode of the carrier, and correction carried out with a sufficiently large interval $T \gg \tau$ at the arrival instants of measurements.

In the general case, for processes described by

$$\mathbf{x}(k) = \mathbf{\Phi}(k, k-1)\mathbf{x}(k-1) + \boldsymbol{\xi}_x(k-1) \tag{6.2}$$

under available measurements

$$\mathbf{z}(k) = \mathbf{Q}_z \left[\mathbf{H}(k)\mathbf{x}(k) + \xi_z(k)\right], \tag{6.3}$$

$$\mathbf{Q}_z(k) = \begin{cases} \mathbf{E} & \text{for } k = nT/\tau,\ n = 1, 2, 3, \ldots \\ 0 & \text{for } k \neq nT/\tau, \end{cases}$$

adaptive analog-discrete filtering algorithms [3] yield the estimates

$$\hat{\mathbf{x}}(k) = \mathbf{x}_{\mathrm{e}}(k) + \mathbf{K}_{\mathrm{fa}}(k)\Delta\mathbf{z}(k), \quad \hat{\mathbf{x}}(0) = \mathbf{x}_0, \tag{6.4}$$

$$\Delta\mathbf{z}(k) = \mathbf{z}(k) - \mathbf{Q}_z(k)\mathbf{H}(k)\mathbf{x}_{\mathrm{e}}(k), \tag{6.5}$$

$$\mathbf{x}_{\mathrm{e}}(k) = \mathbf{\Phi}(k, k-1)\hat{\mathbf{x}}(k-1) + \mathbf{u}_{\mathrm{fin}}(k), \tag{6.6}$$

$$\mathbf{u}_{\text{fin}}(k) = \begin{cases} \begin{cases} \mathbf{f}_{\text{pr}}(\Delta\mathbf{z}(k)) & \text{for } k = nT/\tau \text{ if the adaptive correction} \\ & \text{of the prediction results is used} \\ 0 & \text{for } k \neq nT/\tau \end{cases} \\ 0 \quad \text{if the prediction results are not corrected,} \end{cases} \tag{6.7}$$

$$\mathbf{K}_{\text{fa}}(k) = \mathbf{Q}_{\text{int}}(k)\mathbf{D}(k)\mathbf{H}^{\text{T}}(k)\mathbf{D}_z^{-1}(k), \tag{6.8}$$

$$\mathbf{Q}_{\text{int}}(k) = \begin{cases} \begin{cases} \mathbf{f}_{\text{int}}(\Delta\mathbf{z}(k)) & \text{for } k = nT/\tau \text{ if the adaptive correction} \\ & \text{of the residual gains is used} \\ \mathbf{E} & \text{for } k \neq nT/\tau \end{cases} \\ \mathbf{E} \quad \text{if the residual gains are not corrected,} \end{cases} \tag{6.9}$$

$$\mathbf{D}(k) = \begin{cases} [\mathbf{E} - \mathbf{K}_{\text{fa}}(k)\mathbf{H}(k)]\mathbf{D}_{\text{e}}(k) & \text{for } k = nT/\tau \\ \mathbf{D}_{\text{e}}(k) & \text{for } k \neq nT/\tau, \end{cases} \quad \mathbf{D}(0) = \mathbf{D}_0, \tag{6.10}$$

$$\mathbf{D}_{\text{e}}(k) = \mathbf{\Phi}(k, k-1)\mathbf{D}(k-1)\mathbf{\Phi}^{\text{T}}(k, k-1) + \mathbf{D}_x(k-1). \tag{6.11}$$

Here, $\mathbf{\Phi}$ is the internal connection matrix of (6.2); $\mathbf{Q}_z(k)$ is the matrix of measurement arrival signs; $\mathbf{D}_x$ is the variance matrix of the state noises $\boldsymbol{\xi}_x$ of (6.2); $\mathbf{D}_z$ is the variance matrix of the measurement noises $\boldsymbol{\xi}_z$ of (6.3); $\mathbf{D}$ is the matrix of estimation errors; $\mathbf{u}_{\text{fin}}$ is the prediction correction determined by analyzing the residual $\mathbf{f}_{\text{pr}}(\Delta\mathbf{z}(k))$; $\mathbf{Q}_{\text{int}}$ is the matrix of weights used to correct the residual gain automatically by analyzing $\mathbf{f}_{\text{int}}(\Delta\mathbf{z}(k))$; finally, $\mathbf{E}$ denotes an identity matrix of appropriate dimensions.

The difference between (6.3)–(6.11) and the typical Kalman algorithm lies in two features as follows. The first is that the state extrapolation (6.6) and the computation of the covariance matrix of the prediction errors (6.11) are carried out with a small interval τ whereas the measurements (6.3) and the correction of the estimates $\hat{\mathbf{x}}$ (6.4) with a large interval $T \gg \tau$. Moreover, the second feature predetermines the possibility of using many adaptation techniques and non-simultaneously arriving measurements.

Formulas (6.2)–(6.11) present the two most efficient methods for preventing the divergence of the filtering algorithm (6.3), (6.4) in the case of intensively maneuvering HSA. The first method is based on forming the adaptive prediction correction (6.7); the second one involves the correction (6.9) of the residual gain (6.8). The preference for one of these adaptation schemes depends on the dimension of the target motion model and the set of measurement sensors.

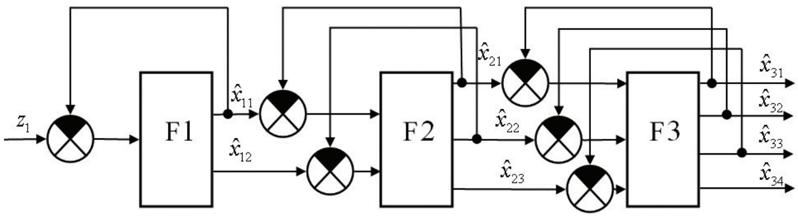
The procedures for calculating the corrections (6.7) and (6.9), including the case of non-simultaneously arriving measurements, were considered in detail in [3].

Note that in the intervals between the arrival of the measurements (6.3), the interceptor control signal is generated based on the prediction results (6.6) for $\mathbf{u}_{\text{fin}} = 0$. During this time, the prediction error is accumulated due to the mismatch between the HSA real flight and its model (6.2). Two operations are executed when the measurements (6.3) from any sensor arrive at the instants corresponding to $n = 1, 2, 3, \ldots$.

The first operation is to calculate the adaptive corrections (6.7) of the prediction (6.6) by changing the residual (6.5) or the correction (6.9) that tunes its weight; the calculation rules were described in [3, 28].

The second operation is intended to generate the estimate (6.4) under the adaptation method selected. In this case, the real estimation accuracy is somewhat worse than the potential one (6.10), (6.11), but the estimates are stable for HSA with complex maneuvers.

In addition to the adaptive prediction correction (6.7) or the correction (6.9) of the residual gain, discussed in [4], a rather effective filtering method is to identify the parameters $\mathbf{\Phi}(k, k-1)$ (6.2)



The functional diagram of the three-stage filter of the fourth order.

of the original state model. However, while providing an efficient adaptation of the model to the application conditions, this method has significantly higher computational costs [28].

Estimating the range, approach speed, and its derivatives from the independent measurements of range (the delay time of reflected signals) and speed (the Doppler frequency) is not difficult. At the same time, a rather complex problem is to estimate relative bearing, LoS angle rate, and its derivatives from the measurements of angles only.

Here, one of the simplest solutions is multistage filtering [4]. The measurements are supplied to the input of a multistage filter. It represents a set of serially connected filters of ascending dimension ($n \geqslant 2$) in which a current filter generates estimates used in the next filter as measurements. As a result, the number of feedback loops increases, thereby improving the stability and accuracy of the derivatives estimates.

The operation principles of this method are explained on the example of a three-stage filter of the fourth order and one measurement sensor. Its functional diagram is presented in the figure with the following notations: F1 is the filter's first stage, which forms the estimates $\hat{x}_{11}$ and $\hat{x}_{12}$ from the measurement z_1 and passes them to the second stage as measurements; F2 is the filter's second stage, which forms the estimates $\hat{x}_{21}, \hat{x}_{22}$, and $\hat{x}_{23}$ and passes them to the third stage as measurements; F3 is the filter's third stage, which forms the estimates $\hat{x}_{31}, \hat{x}_{32}, \hat{x}_{33}$, and $\hat{x}_{34}$ and passes them to the user.

The effectiveness of the multistage filtering method was validated in [4], where the angle and its derivatives up to the fourth order were stably estimated as an illustrative example with a single measurement sensor.

7. CONCLUSIONS

The material presented in this paper leads to the following findings.

The requirements for ways to optimize guidance methods to HSA [26], including the need to design time-varying control laws in a given area of application conditions under current constraints and implementability conditions, and the results of previous studies have been considered as a basis to assess the capabilities of various optimization methods for designing interceptor's control laws.

In particular, the following approaches have been assessed:

- the classical optimal control theory in the Letov–Kalman formulation;
- local optimization methods, including those with real and virtual disturbances;
- local optimization by quadratic-biquadratic performance functionals;
- the concept of inverse dynamics problems;
- ways to provide the information support of guidance methods designed.

According to the comparative analysis results, the local optimization method by the minimum of quadratic-biquadratic performance functionals and the method based on inverse dynamics problems have the best capabilities according to the set of requirements.

In future papers, we will assess the capabilities of gradient-based optimization and the so-called intelligent control methods.

In addition, examples of designing particular guidance methods within the most appropriate optimization approaches will be presented.

In conclusion, an important aspect should be emphasized. When several optimization methods are used, the problem of a qualified choice of the best result arises inevitably. In the simplest case, this choice is made by comparing performance and survivability indicators. The so-called foresight concept [29] can be adopted for a more justified choice. This concept allows automating the choice of the best alternative based on many heterogeneous tactical, economic, and technological attributes.

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